Physics of the alphorn

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Pipes and frequencies

To study the physical backgrounds of the acoustical properties of the alphorn we have to go back to the theory of standing sound waves in pipes, in particular conical pipes.
First we repeat 4 well-known models in this theory, Then, in the next section, we will investigate which of these models applies to the acoustics of the alphorn and to what extent.

Model 1:

<u>Open cylindrical</u> pipe, with length L

Vibrating of the air column admits the following frequencies:

f, 2**f**, 3**f**, 4**f**, 5**f**, ... where **f** = **v** / 2**L**

v is the speed of sound: **v** = $331(1 + t / 273)^{1/2}$ (m/s under **t** degrees Celsius).
t = 15: v = 340,0
t = 20: v = 342,9

<u>Physical explanation</u>
Based on elementary physics, by studying the possible node and anti-node patterns for the sound wave.

Model 2:

<u>Closed cylindrical</u> pipe (i.e. closed at one end), with length L

Vibrating of the air column admits the following frequencies:

f, 3**f**, 5**f**, 7**f**, ... where **f** = **v** / 4**L**

<u>Physical explanation</u>
Also elementary, like above.

Model 3:

<u>Conical</u> pipe, with length L

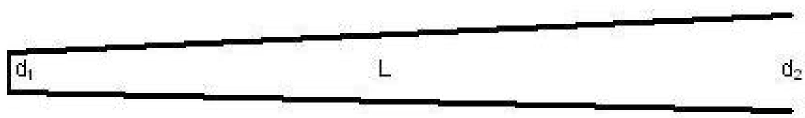
Vibrating of the air column admits the following frequencies:

f, 2**f**, 3**f**, 4**f**, 5**f**, ... where **f** = **v** / 2**L**

<u>Physical explanation</u>
Less simple. Can be shown by solving the wave equation.

Model 4:

<u>Truncated conical</u> pipe, with length L and inner diameters d_1 and d_2



Vibrating of the air column admits the following frequencies:

$\mathbf{f}_1$, $\mathbf{f}_2$, $\mathbf{f}_3$, $\mathbf{f}_4$, $\mathbf{f}_5$, ... with

$$f_n = (n - \tfrac{1}{2})\frac{c}{4L'}\left\{1 + \left[1 + \frac{4(d_2 - d_1)}{\pi^2 d_1 (n - \frac{1}{2})^2}\right]^{1/2}\right\}$$

(formula 2 of <u>Neville Fletcher</u>)

where **c** is the speed of sound and **L'** = **L** + 0,3 $\mathbf{d}_2$, the so-called <u>acoustic length</u>.
0,3 $\mathbf{d}_2$ is the <u>end-correction</u> at the open end of the pipe.

<u>Condition</u>
The apex a = d_1L / (d_2-d_1) is not to small. If a is to small, then model 3 applies.

<u>Physical explanation</u>
Not simple. See: *N.H. Fletcher and T.D. Rossing - The Physics of Musical Instruments, 1991.*

<u>Particular case</u>: $d_1 = d_2$
Then f_n = 2 (n-½) c / 4L' = (2n-1) c / 4L'. This is model 2, with L replaced by L'.

<u>The relationship between the models 2, 3 and 4</u>

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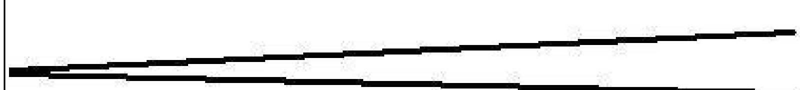
<tr><td>If a is to small (a → 0), then <u>model 3</u> (not a special case of model 4) applies
</td></tr>
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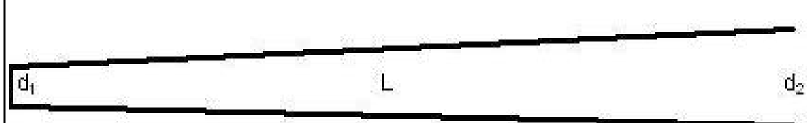
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If $a \to \infty$ (i.e. $d_1 \to d_2$), then model 2 (special case of model 4) applies

Application to the alphorn

We will now investigate how useful the previous models are to explain the frequencies produced by an alphorn.
For this purpose I have chosen my own alphorn, in E, build by myself.
Specifications:

> Length of the conical tube included the mouthpiece: 337 cm
> Inner diameters of the conical tube: 1,3 cm and 5,5 cm.
> Length of the bell: 60 cm.
> Inner diameter at the end of the bell: 23 cm.

Anyhow, an alphorn is neither a complete cone nor a truncated cone! However, if we put aside the bell of the alphorn, then we get a truncated conical tube and this tube we can test.
Specifications of this truncated conical tube:

> Length L: 337 cm
> Diameters d_1 and d_2: 1,3 cm and 5,5 cm

$a = d_1L / (d_2-d_1) = 1,3 \times 337 / (5,5 - 1,3) \approx 104$ (cm), so the apex a is not small and therefor we may expect that model 4 will better apply than model 3.

To keep the testing process simple, I made use of a chromatic tuner to identify the produced notes.

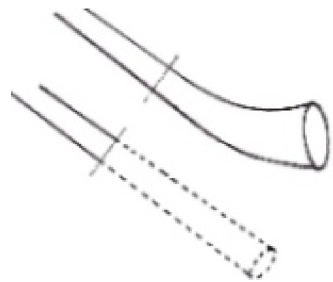
Results of this test:

Frequencies according to model 4 L = 337 $d_1 = 1,3$ $d_2 = 5,5$	Frequencies according to model 3 L= 337 $d_1 = 0$ $d_2 = 5,5$ f = 343 / 2L'	Produced frequencies
$f_1 = 44$ (Hz)	f = 51 (Hz)	
$f_2 = 86$	2f = 101	84 (Hz)
$f_3 = 133$	3f = 152	132
$f_4 = 182$	4f = 202	179
$f_5 = 232$	5f = 253	230
$f_6 = 282$	6f = 304	280
$f_7 = 332$	7f = 354	332
$f_8 = 382$	8f = 405	378
$f_9 = 432$	9f = 455	434
$f_{10} = 483$	10f = 506	481

f_{11} = 533	11f = 557	525
f_{12} = 584	12f = 607	572

The results are clear. Model 4 gives a good prognosis for the produced notes. The frequencies of model 3 are too high.

This result enables us to conclude that model 4 will also give a good prognosis for the produced notes, is we should extend the tested truncated conical tube with length 337 cm to a truncated conical tube with length 397 cm.
After that we will be in the position to compare these notes with the real notes of our alphorn in E and to see what the effect of the bell is.



An easy calculation shows that the diameter at the end of the extended truncated cone is 6,25 cm.

Results of this comparison:

Extended truncated cone (no bell) L = 397 d_1 = 1,3 d_2 = 6,25 Frequencies according to model 4	Alphorn in E (with bell) L = 397 Produced frequencies	 2[e] column can be read as
f_1 = 40 (Hz)		
f_2 = 74	80	2f
f_3 = 114	122	3f
f_4 = 155	165	4f
f_5 = 197	210	5f
f_6 = 239	252	6f
f_7 = 282	290	7f
f_8 = 325	330	8f
f_9 = 367	371	9f
f_{10} = 410	412	10f
f_{11} = 453	450	11f
f_{12} = 496	492	12f

The second and third column show the well-know effect of the bell:
the bell raises progressively the lower frequencies.
The result is a full harmonic series with the fundamental missing!

Conclusions

- In the first instance we are inclined to conclude that it is the combination of model 4 and the bell effect that makes that blowing the alphorn (with mouthpiece) results in a full harmonic series, with the fundamental missing. But, the fact that we used a mouthpiece in the previous investigations cannot be neglected. We have to assume that the well-known effect of the mouthpiece has also its influence on the sounding frequencies. **The mouthpiece forces the upper resonances progressively down.** I think the used measuring-method is not enough precise to show this effect clear.

- The sounding frequencies of the alphorn are:

  2**f**, 3**f**, 4**f**, 5**f**, ...

  The formula 2**f** = **v** / **L'** proves to be a reasonable approximation of the second natural mode of the alphorn.
  In the case of our alphorn in **E** this formula gives:
  $2f = v / L' = v / (L + 0,3\ d_2) = 343 / (3,97 + 0,3 \times 0,23) = 84,9$ (Hz), which equals almost the value 82,4 (Hz) of the pitch E2.

- Now we see how strong the suggestion is that model 3 is a correct physical model for the alphorn! Unfortunately, this interpretation is not tenable. The physical backgrounds are more complicated.
  A more serious fallacy is to see model 1 as a physical model for the alphorn. This cannot be the case, since the mouthpiece end of the tube behaves acoustically like a closed end. At this end the pressure variations are not zero but a maximum.

- We have seen that the applicability of the four models is limited. Only model 4 (with formula 2 of Neville Fletcher) gives a good, but only partial contribution to the understanding of the acoustical working of the alphorn. The mouthpiece effect and the bell effect play a significant and indispensable role. This role is well-known with respect to the acoustics of the trumpet.

References

John Backus - The Acoustical Foundations of Music, 1969
Arthur H. Benade - Fundamentals of Musical Acoustics, 1976
Neville H. Fletcher, Thomas D. Rossing - The Physics of Musical Instruments, 1991